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What can we learn from the 126 GeV Higgs boson for the Planck scale physics ?
- Hierarchy problem and the stability of the vacuum - *

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The discovery of the Higgs particle at around 126 GeV has given us a big hint towards the origin of the Higgs potential. The running quartic self-coupling decreases and crosses zero somewhere in the very high energy scale. It is usually considered as a signal of the instability of the standard model (SM) vacuum, but it can also indicate a link between the physics in the electroweak scale and the Planck scale. Furthermore, the LHC experiments as well as the flavor physics experiments give strong constraints on the physics beyond the SM. It urges us to reconsider the widely taken approach to the physics beyond the SM (BSM), namely the approach based on the gauge unification below the Planck scale and the resulting hierarchy problem. Motivated by the recent experiments, we first revisit the hierarchy problem and consider an alternative appoach based on a classical conformality of the SM without the Higgs mass term.

In this talk, I review our recent proposal of a B-L extension of the SM with a flat Higgs potential at the Planck scale [1, 2]. This model can be an alternative solution to the hierarchy problem as well as being phenomenologically viable to explain the neutrino oscillations and the baryon asymmetry of the universe. With an assumption that the Higgs has a flat potential at the Planck scale, we show that the B-L symmetry is radiatively broken at the TeV scale via the Coleman-Weinberg mechanism, and it triggers the electroweak symmetry breaking through a radiatively generated scalar mixing. The ratio of these two breaking scales is dynamically determined by the B-L gauge coupling.

I. CENTRAL DOGMA OF PARTICLE PHYSICS

In the LHC era, we acquired various hints towards the physics beyond the SM. The first hint is of course the mass of the recently discovered Higgs-like particle. The value of 126 GeV is quite interesting because it is close to the boarder of the stability bound. Given the vev of the Higgs at 246 GeV, its mass gives an information of the curvature of the potential at the minimum. As the mass 126 GeV is smaller than 246 GeV, the Higgs potential is rather shallow and unstable against the radiative corrections. The (in)stability of the SM vacuum can be investigated explicitly by looking at the running bahaviour of the quartic coupling. The beta function of the quartic Higgs coupling λ_H is given by

$$\beta_H = \frac{1}{16\pi^2}\left(24\lambda_H^2 - 6Y_t^4 + \frac{9}{8}g^4 + \frac{3}{8}g_Y^4\right). \tag{1}$$

Y_t is the top Yukawa coupling and g, g_Y are $SU(2)_L, U(1)_Y$ gauge couplings. It is either positive or negative whether the negative contribution by the large top Yukawa coupling is compensated by the gauge couplings and the Higgs quartic coupling. The corresponding quartic coupling to the 126 GeV Higgs boson does not suffice to compensate it and the beta function is negative. So the running quartic coupling crosses zero somewhere at the UV energy scale. It is very suggestive that the observed value of the Higgs mass is close to the stability bound up to the Planck scale [3]

$$M_h[\text{GeV}] > 129.2 + 1.8\left(\frac{M_t[\text{GeV}] - 173.2}{0.9}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{th}. \tag{2}$$

When the Higgs mass is lighter than the above bound, new physcis must appear below the Planck scale. But if it lies just on the border of the stability bound, it gives a big hint to the origin of the Higgs potential at the Planck scale[4].

Another important information is that the LHC results are almost consistent with the SM. Furhtermore the precision experiments of the flavor physics, Babar, Belle and LHCb, gave stringent constraints on the physics beyond the SM. Of course, in spite of the above rather unexpectedly good agreement with the SM, there exist phenomena which cannot be explained within the SM. Nuetrino oscillation requires the dimension 5 operator $l\phi l\phi$ and a new scale beyond the SM must be introduced. The baryon asymmetry of the universe also requires an additional source of the CP violation. The SM anyway needs to be extended to explain these phenomana.

The most common appoach to go beyond the SM is based on a unification of the gauge couplings below the Planck scale, i.e. the GUT scale. Then we need a natural explanation why the electroweak scale is much smaller than the GUT scale. In order to solve the hierarchical structure of the scales, the supersymmetry is introduced. Here I call the sequence of ideas from GUT to the hierarchy problem and the low energy supersymmetry *the central dogma of particle physics*. In addition to solving the hierarchy problem, it can improve the gauge coupling unification as well as providing candidates of the dark matter particles. But as the bonus we get or as the price we pay, it predicts many new particles at the TeV scale and the recent experiments have given strong constraints on the models with low energy supersymmetry.

In such circumstances, it may be a good time to reconsider the central dogma of particle physics. In this note, we take an approach to the hierarchy problem suggested by Bardeen. In the next section, we interpret the Bardeen's argument in terms of the renormalization group. If we adopt the argument, the most natural mechanism to break the electroweak symmetry is the Coleman-Weinberg (CW) mechanism. But we know that the CW mechanism does not work within the SM because of the large top Yukawa coupling, so we need to extend the SM. In section 3, we introduce our model, a classically conformal $B - L$ extension of the SM and then discuss the dynamics of the model.

II. BARDEEN'S ARGUMENT OF THE HIERARCHY PROBLEM

We pay a special attention to the almost scale invariance of the SM. At the classical level, the SM Lagrangian is conformal invariant except for the Higgs mass term. Bardeen argued [5] that once the classical conformal invariance and its minimal violation by quantum anomalies are imposed on the SM, it may be free from the quadratic divergences.

Bardeen's argument on the hierarchy problem may be interpreted as follows [6]. We classify divergences of the scalar mass term in the SM into the following 3 classes,

- quadratic divergences: Λ^2

- logarithmic divergences with a small coefficient: $m^2 \log(\Lambda/\mu)$

- logarithmic divergences with a large coefficient: $M^2 \log(\Lambda/\mu)$

The logarithmic divergences are operative both in the UV and the IR. In particular, it controls a running of coupling constants and is observable. On the other hand, the quadratic divergence can be always removed by a subtraction. Once subtracted, it no longer appears in observable quantities. In this sense, it gives a boundary condition of a quantity in the IR theory at the UV energy scale where the IR theory is connected with a UV completion theory. Indeed, the RGE of a Higgs mass term m^2 in the SM

$$V(H) = -m^2 H^\dagger H + \lambda_H (H^\dagger H)^2 \tag{3}$$

is approximately given by

$$\frac{dm^2}{dt} = \frac{m^2}{16\pi^2} \left(12\lambda_H + 6Y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2 \right). \tag{4}$$

The quadratic divergence is subtracted by a boundary condition either at the IR or UV scale. Once the initial condition of the RGE is given at the UV scale, it is no longer operative in the IR. The RGE shows that the mass term m^2 is multiplicatively renormalized. If it is zero at a UV scale M_{UV}, it continues to be zero at lower energy scales. In this sense, the quadratic divergence is not the issue in the IR effective theory, but the issue in the UV completion theory. Hence if the SM (and its extension at the TeV scale) is directly connected with a UV completion theory at the Planck scale physics, the hierarchy problem turns out to be a problem of the boundary

condition at the UV scale. If the UV completion theory is an ordinary field theory, it will be difficult to protect the masslessness of the scalar particle against radiative corrections by massive particles of the UV scale unless we introduce, e.g. the low-energy supersymmetry. But in the string theory, symmetry is sometimes enhanced on a moduli space and massless scalars can survive even without supersymmetry. Also discrete symmetry like T-duality, which is invisible in the low energy effective theory, may prohibit a generation of potential at the string scale.

The multiplicative renormalization of the Higgs mass term is violated by a mixing with a massive field in the loop. If the massive field aquires its mass in a different mechanism with the EWSB, the Higgs mass has a logarithmic divergence

$$\delta m^2 \sim \frac{\lambda^2 M^2}{16\pi^2} \log(\Lambda^2/m^2) \tag{5}$$

which modifies the RGE as

$$\frac{dm^2}{dt} = \frac{m^2}{16\pi^2}\left(12\lambda_H + 6Y_t^2 - \frac{9}{2}g^2 - \frac{3}{2}g_Y^2\right) + \frac{M^2}{8\pi^2}\lambda^2. \tag{6}$$

The last term corresponds to the logarithmic divergence with a large coefficient. The coefficient M^2 has nothing to do with the mass of the Higgs m^2, and it violates the multiplicativity of the Higgs mass. Thus the hierarchy problem, namely the stability of the EWSB scale, is caused by such a mixing of relevant operators (mass terms) with hierarchical energy scales $m \ll M$. In the Bardeen's argument, he also imposes an absence of intermediate scales above the EW scale. The logarithmic divergence with a large coefficient (5) is sometimes confused with the quadratic divergence, but if the UV completion theory is something like a string theory, they should be distinguished.

From the above considerations, the hierarchy problem can be solved by imposing the following two different conditions;

- Correct boundary condition at the UV (Planck) scale M_{pl}

- Absence of mixings in intermediate scales below M_{pl}

The first condition subtracts the quadratic divergence at the Planck scale. It must be solved in the UV completion theory such as the string theory. The most natural boundary condition is that scalar fields which appear in the low energy physics are massless at the Planck scale. On the other hand, the second condition assures the absence of logarithmic divergences with large coefficients. Even if the scalars are massless at the Planck scale, they receive large radiative corrections from the mixing with other relevant operators. Without a cancellation mechanism like the supersymmetry, we need to impose an absence of intermediate scales between EW (or TeV) and Planck scales. Hence all symmetries are broken either at the Planck scale or near the EW scale. Especially, the breakings of the supersymmetry or the grand unification of gauge coupling should occur at the Planck scale. This second condition is also emphasized in the Bardeen's argument [5]. In such a scenario, Planck scale physics is directly connected with the electroweak physics [8].

Hence a natural boundary condition of the mass term at the UV cut-off scale, e.g. M_{Pl}, is

$$m^2(M_{Pl}) = 0. \tag{7}$$

This is the condition of the classical conformality of the BSM. The condition (7) must be justified in the UV completion theory, and from the low energy effective theory point of view, it is just imposed as a boundary condition [14].

III.   FLAT POTENTIAL AT THE PLANCK SCALE

The condition (7) restricts the form of the Higgs potential as

$$V(H) = \lambda_H (H^\dagger H)^2. \tag{8}$$

Here λ_H is the running coupling and the RG improved effecitve potential is given by making the coupling $\lambda_H(H)$ field dependent. The mass term is not generated even in the IR as discussed in the previous section once the boundary condtion (7) is imposed at the boundary with the UV completion theory. The mass of the Higgs at 126 GeV suggests that the running coupling becomes asymptotically vanishing near the Planck scale.

The current bounds (2) is a bit heavier than the experimental data, but in this note, we assume that the Higgs quartic coupling vanishes at the Planck scale. Hence

$$V(H) = 0 \text{ at the Planck scale.} \tag{9}$$

The condition may connect the SM in the IR with the string theory in the UV. Now we have to solve two problems. The first is whether we can construct a phenomenologically viable model starting from the condtion of the flatness of the Higgs potential (9), and the second is to derive such a boundary condtion from the UV completion theory such as a string theory. Supersymmetry or grand unification, if exists, are broken at the Planck scale. In the following we focus on the first problem by proposing a B-L extension of the SM with a flat potential at the Planck scale. The second issue is left for future investigations.

Since the IR theory is assumed to have the boundary condition (9), the electroweak symmetry breaking should occur radiatively, namely the Coleman-Weinberg mechanism. However, it is now well-known that the CW mechanism cannot occur within the SM because of the large top-Yukawa coupling. Indeed, the CW mechanism is realized only when the beta-function of the quartic scalar coupling is positive and the running quartic coupling crosses zero somewhere in the IR. But as we saw, the beta function of the quartic Higgs coupling is positive in the SM and its behavior is opposite to the CW mechanism. Hence, in order to realize the EWSB, we need an additional sector in which the symmetry is broken radiatively by the CW mechanism and whose symmetry breaking triggers the EWSB. In the next section, we introduce our model, namely a B-L extension of the SM with a flat potential at the Planck scale.

IV. B-L EXTENSION OF THE SM WITH FLAT POTENTIAL AT PLANCK

The idea to utilize the CW mechanism to solve the hierarchy problem was first modelled by Meissner and Nicolai [7] (see also [9]). In addition to the SM particles, they introduced right-handed neutrinos and a SM singlet scalar Φ. Inspired by the work, we proposed a minimal phenomenologically viable model [2]. It is the minimal B-L model [10], but with a classical conformality. The model is similar to the one proposed by Meissner and Nicolai [7], but the difference is whether the B-L symmetry is gauged or not. In a recent paper we further showed that by imposing the flatness (9) of the Higgs potential at the Planck scale the B-L breaking scale is related with the EWSB scale. The ratio of two scales is dynamically determined by the B-L gauge coupling and the B-L breaking scale is naturally constrained to be around TeV scale [11] for a not so small B-L gauge coupling.

Besides the SM particles the model consists of the B-L gauge field with the gauge coupling g_{B-L}, right-handed nuetrinos ν_R^i ($i = 1, 2, 3$ denotes the generation index) and a SM singlet complex scalar field Φ with two units of the B-L charge. The model is anomaly free. The Lagrangian contains Majorana Yukawa coupling $\sim Y_N^i \Phi \bar{\nu}_R^{ic} \nu_R^i$, and the see-saw mechanism gives masses to the left-handed neutrinos once the scalar Φ acquires vev.

V. SYMMETRY BREAKINGS OF B-L AND EW

Since the B-L gauge symmetry is broken by the CW mechanism, the breaking scale is correlated with the quartic coupling λ_Φ at the UV scale. Its running is described by

$$\frac{d\lambda_\Phi}{dt} = \frac{1}{16\pi^2} \left(20\lambda_\Phi^2 - \frac{1}{2} Tr\left[Y_N^4 \right] + 96 g_{B-L}^4 + \cdots \right). \tag{10}$$

If the Mayorana Yukawa coupling is not so large, the beta function is positive. The typical behavior of the running λ_ϕ is drawn in Fig. 1. It crosses zero at a lower energy scale M_0, then the B-L symmetry is broken at $M_{B-L} \sim M_0 \exp(-1/4)$ through the CW mechanism[1].

As shown in the paper [2], the ratio of the scalar boson mass to the B-L gauge boson mass is given by

$$\left(\frac{m_\Phi}{m_{Z'}} \right)^2 \sim \frac{6}{\pi} \alpha_{B-L} \ll 1. \tag{11}$$

The condition that the B-L gauge coupling does not diverge up to the Planck scale requires $\alpha_{B-L} < 0.015$ at M_{B-L}. Hence the scalar boson becomes lighter than the B-L gauge boson, $m_\Phi^2 < 0.03 \, m_{Z'}^2$. Such a very light scalar boson is a general prediction of the CW mechanism.
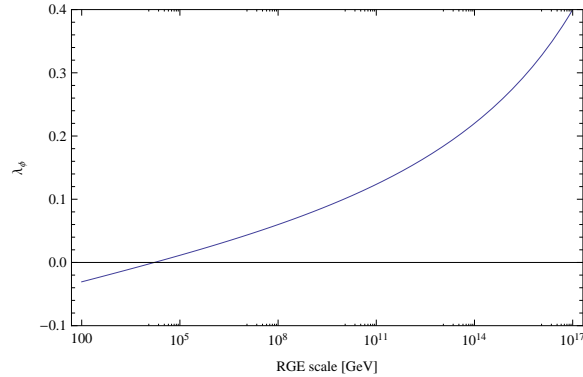
FIG. 1: RG evolution of the self-coupling λ_ϕ of a SM singlet scalar ϕ. Since the β function is positive, the running coupling crosses zero at a lower energy scale.

The EWSB is triggered by the B-L breaking. The flatness condition (9) of the Higgs potential predicts an absence of the scalar mixing at the Planck scale. Hence B-L and EW sectors are decoupled each other in the UV. But since the matter fields are coupled to both $U(1)_Y$ and $U(1)_{B-L}$, these two sectors become mixed through the $U(1)$-mixing. As a result, the scalar mixing term $\lambda_{mix}(H^\dagger H)(\Phi^\dagger \Phi)$ appears in the IR. It is interesting that a very small negative mixing λ_{mix} is always generated irrespective of the details of other parameters once we assume the flatness condition $\lambda_{mix}(M_{pl}) = 0$. By solving the RGE [1] we showed that the scalar mixing term is dynamically generated around $\lambda_{mix} \sim -4 \times 10^{-4}$. If the Φ field acquires a VEV $\langle\Phi\rangle = M_{B-L}$, the mixing term $\lambda_{mix}(H^\dagger H)(\Phi^\dagger \Phi)$ gives an effective mass term of the Higgs field. Since the coefficient λ_{mix} is negative, the EWSB is triggered and the Higgs VEV is given by

$$v = \langle H \rangle = \sqrt{\frac{|\lambda_{mix}|}{\lambda_H}} M_{B-L} \tag{12}$$

This gives a ratio between the EWSB scale to the B-L symmetry breaking scale. The scalar mixing is determined in terms of the gauge couplings, so the ratio of two breaking scales is also determined dynacamilly in terms of the gauge coupling g_{B-L}.

VI. MODEL PREDICTIONS

The dyanamics of the model is controlled by two parameters, g_{B-L} and λ_Φ, which determines the two breaking scales of B-L and EW. The experimental input $v = 246$ GeV gives a relation between these two and the dynamics of the model is essentially described by a single parameter. The figure 2 shows the prediction of our model. The vertical axis is the strength of α_{B-L} and the horizontal axis is the mass of the B-L gauge boson. The black line (from top left to down right) shows the prediction of our model. If an extra $U(1)$ gauge boson and a SM singlet scalar are found in the future, the prediction of our model is the mass relation (11), e.g., $m_\phi \sim 0.1\ m_{Z'}$ for $\alpha_{B-L} \sim 0.005$. The CW mechanism in the B-L sector predicts a much lighter SM singlet Higgs boson than the extra $U(1)$ gauge boson. It is different from the ordinary TeV scale B-L model where the symmetry is broken by a negative squared mass term.

Nuetrino oscillation is realized by the type I see-saw mechanism with small neutrino Yukawa couplings. Baryon number asymmetry of the universe may be generated through the TeV scale leptogenesis with almost degenerate Majorana masses[12]. Furhter phenomenological issues such as $U(1)$ mixing or the lepton number violation at the TeV scale are discussed in a separate paper.

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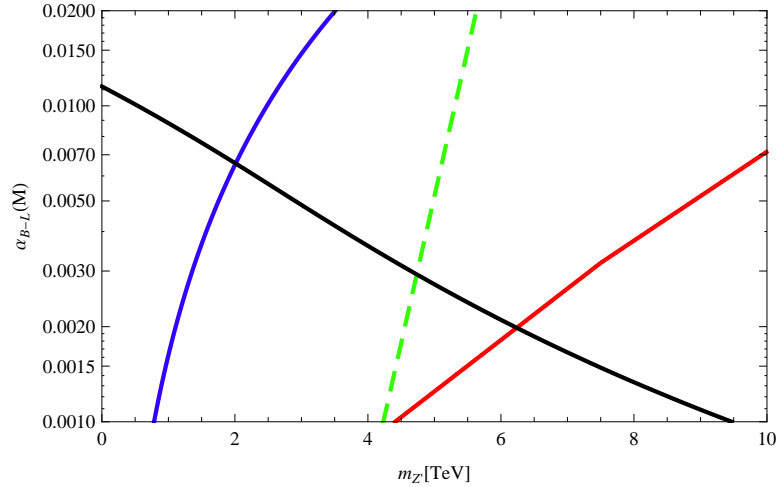
FIG. 2: Model prediction is drawn in the black line (from top left to down right). The $B - L$ gauge coupling α_{B-L} and the gauge boson mass $m_{Z'}$ are related because of the flat potential assumption at the Planck scale. The left side of the most left solid line in blue has been already excluded by the LEP experiment. The left of the dashed line can be explored in the 5-σ significance at the LHC with $\sqrt{s}$=14 TeV and an integrated luminosity 100 fb^{-1}. The left of the most right solid line (in red) can be explored at the ILC with $\sqrt{s}$=1 TeV, assuming 1% accuracy.

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[14] The condition (7) may look similar to the Veltman condtion [13], but they are conceptually different at all. In the Veltman condition, the quadratic divergence is considered to be cancelled between various contributions of bosons and fermions. Such a cancellation occurs in a very special situation of the IR physics. On the contrary, the condition (7) is independent of the matter content in the IR, and robust against a change of scales.